APPLICATION OF MATHEMATICAL MODEL
FOR APIARIES LOCATION EVALUATION

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Abstract. An approach for evaluation of apiaries' location in flat-hilly regions by using a mathematical model is presented. The study was conducted in 2021 based on the assessment of the existing bee forage resources as Black locust (*Robinia pseudoacacia*), Linden (*Tilia cordata*), Sunflower (*Helianthus annuus*), Rapeseed (*Brassica napus*), Lavender (*Lavandula*) and the number of bee colonies kept in three different regions in the North-eastern part of Bulgaria. We use the mathematical model designed to estimate the potential location where hives can be placed, comparable to a coefficient characterizing the extent to which this place is desired to be created. The evaluation coefficient named ρ_i , preferably in a given place, the sum of the "values" of all feeding places in relation to the given apiary place are taken in consideration. Possible solutions are illustrated with numerical values giving assessment of potential distribution sites for cases without overpopulation of the area with bee colonies and for areas with overpopulation. The effectiveness of the proposed approach is reported in two directions. On the one hand, the benefit for beekeepers in their selection of suitable places for apiaries guaranteeing optimal nutrition of bee colonies. The correct assessment of the location will provide pollen and nectar for bees, reduction of harmful effects of overpopulation with bee colonies and high yields of bee products. On the other hand, the benefit for farmers growing flowering agricultural crops in the respective area is expressed in guaranteeing better pollination of crops, which is a prerequisite for higher yields.

Keywords: bee colonies, apiaries location, overpopulation, assessment.

Introduction

To maintain the vital functions of bee colonies and the production of commodity honey, it is important to have sufficient foraging resources of nectar and pollen [1]. During natural reproduction, the abandoned swarm settles at a certain distance from the colony from which it came out, as the migration behaviour of the colony was assumed to follow the optimal foraging theory. Some authors [2] present a conceptual model to determine, as well as to predict, the possible migratory route of a colony of bounded-rational species. Unfortunately, in artificial reproduction, newly established colonies are often placed in the same apiary or at close range. This creates competition for food between the colonies in the apiary, as well as competition between individual apiaries. Finding suitable places for apiaries is a very difficult task for all beekeepers. It is influenced by the changing location of agricultural areas, the changing number of bee families in the region over the years, regulations governing beekeeping, theft of beehives outside the settlements, the impact of changing meteorological factors on nectar production during flowering and so on. However, beekeepers choose places for apiaries, prioritizing their own preferences, such as the ability to easily service the apiaries and the infrastructure built next to them, the possibility of monitoring and safety of the apiary, the presence of forest and cultural flowering plants nearby, etc. In the modern beekeeping in flat-hilly regions in Bulgaria there is a rapid increase in the number of bee colonies in areas rich in flowering vegetation, which leads to their overpopulation. From our study conducted in 2019 for Ruse region we found that in 83% of the cases we have overpopulation with apiaries. This creates competition between bee colonies leading to reduced yields of bee products, weak and unproductive bee colonies, danger of diseases and pests. On the other hand, in some areas there is a reduced number of bee colonies, which has an adverse effect on the pollination of flowering crops. Some publications analyze the impacts of honeybee density on crop yield [3]. Optimizing the location of bee colonies and areas with flowering vegetation is a prerequisite for sustainable agriculture.

The search for optimal solutions to reduce overpopulation has been suggested by [4], who is trying to solve the problem of overpopulation at stingless bees (*Trigona biroi Friese*) using optimization by choosing the best element from some set of available alternatives. Other interesting solutions have been suggested by [5], where the mathematical models are grouped into two sections considering the following scenarios 1) minimizing competition among foragers due to limited food sources, and 2) maximizing the use of the species as pollinators given small number of foragers. In another publication [6] in order to maximize production of honey and minimize unhealthy competition among foraging bees the authors have considered a weighted sum model and analytic hierarchical model. Other researchers [7] present a model for finding the number of honey bee colonies needed for the optimal foraging process

in the specific location taking into account potential field productivity, possible chemical contamination, surroundings of the apiary. Some studies [8] solve an optimization task aimed to optimize the beekeeper's preferences for individual places. The location of the apiary chosen by the beekeeper is not always optimal for the bee colonies. When choosing a place for an apiary, in addition to foraging resources, the beekeeper often takes into account the infrastructure and accessibility to the place, the possibility of service, security and other factors. Given these features, well-constructed mathematical models are multi-criteria, optimizing several criteria simultaneously, the optimal set of Pareto solutions is sought, and then one is chosen [9].

The aim of our study is to evaluate apiaries' location in flat-hilly regions with overpopulation and without overpopulation with bee colonies (*Apis mellifera macedonica*), by using a mathematical model.

Materials and methods

The study was conducted in 2021 based on the assessment of the existing bee forage resources as Black locust (*Robinia pseudoacacia*), Linden (*Tilia cordata*), Sunflower (*Helianthus annuus*), Rapeseed (*Brassica napus*), Lavender (*Lavandula*) and the number of bee colonies kept in three different regions in the Northeast part of Bulgaria. Geographical location of the experimental regions is: for region Brestovica 43º32'4.02"N and 25º45'14.10"E at an altitude of 223 m, for region Yuper 43º54'28.59" N, 26º23'49.02"E at an altitude of 107 m, for region Glavinica 43º54'51.94" N, 26º50'10.14" E, at an altitude of 114 m. In the region of Brestovica forage resources are Black locust (*Robinia pseudoacacia*) 106 ha, Linden (*Tilia cordata*) 67 ha, Sunflower (*Helianthus annuus*) 244 ha, Rapeseed (*Brassica napus*) 142 ha.

In the region of Yuper major honey source plants are Black locust (*Robinia pseudoacacia*) 68 ha, Sunflower (*Helianthus annuus*) 225 ha, Rapeseed (*Brassica napus*) 220 ha. In the region of Glavinica major honey source plants are Black locust (*Robinia pseudoacacia*) 3 ha, Linden (*Tilia cordata*) 13 ha, Sunflower (*Helianthus annuus*) 461 ha, Lavender (*Lavandula*) 15 ha, Honey locust (*Gleditsia triacanthos*) 27 ha.

The total number of bee colonies in Brestovica region is 784, for Yuper region 568, for Glavinica region 1225. All colonies from the experimental regions are housed in Dadant-Blatt hives. The bees are of the species (*Apis mellifera macedonica*).

These five major crops determined the main honeybee pasture in the study area. Other vegetation types are vineyards (*Vitis*), oak (*Quercus*), maize (*Zea mays*), wheat (*Triticum*), barley (*Hordeum vulgare*), soybeans (*Glycine max*). If all the factors remain constant, the productivity of the bee colonies is in correlation with nectar secretion potential of bee forage species and the existing honeybee colony density. It is very important for the proper development of bee colonies to find the best location of a bee hive in the certain study area. The distance between region Brestovitsa and region Yuper is 66.38 km, between region Brestovitsa and region Glavinica it is 96.87 km, between region Yuper and region Glavinica it is 35.31 km. Geo-referenced images of the studied regions are shown in Fig. 1-3.
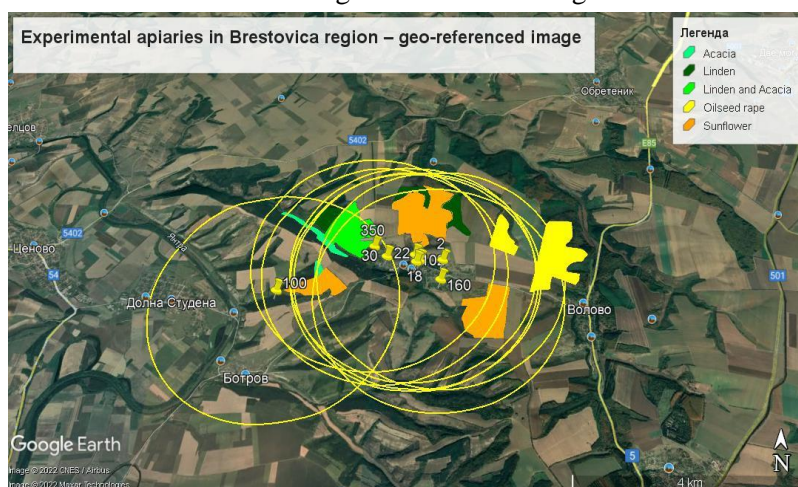

Fig. 1. **Experimental apiaries in Brestovica region – geo-referenced image**

One of the important factors that determine the choice of bees to carry nectar and pollen from the source of feeding to the bee colony is the distance and the energy spent by the bees in flight. The productive flight distance of 2500 m of *Apis mellifera macedonica* is shown with yellow circles. The numbers in the middle of the circle are the number of bee colonies in each apiary.

In determining the expected yield of honey for one year depending on the productive potential of crop rotations, the method presented in [10] was used.

In Europe, a normal-sized colony consumes relatively 60-202 kg of honey per year [11]. Based on this rough estimation and assuming that all factors remain constant, to harvest 1 kg of surplus honey, the colony has to consume a further 1 kg of honey for survival, brood rearing, as fuel energy to foragers. Based on this requirement 2 kg of honey will be required in order to harvest 1 kg of honey and 1 kg of honey for colony maintenance [12].

In our study we use the mathematical model of [13], which is designed to estimate the potential location $M_i, i = 1\ldots m$ where hives can be placed, comparable to the coefficient characterizing the extent to which this place is desired. When constructing this coefficient, the following is take into account: 1) the distance c_{ij} from place M_i to the source of feeding $S_j, j = 1\ldots m$, 2) carrying capacity of the plant cluster $b_j, j = 1\ldots m$ (measured in the number of bee colonies that can feed), 3) competition with other places.

The function f has the following form:

$$f(x) = \begin{cases} -\dfrac{x}{2500} + 1, x \in [0;2500] \\ 0, x \notin [0;2500] \end{cases} \tag{1}$$
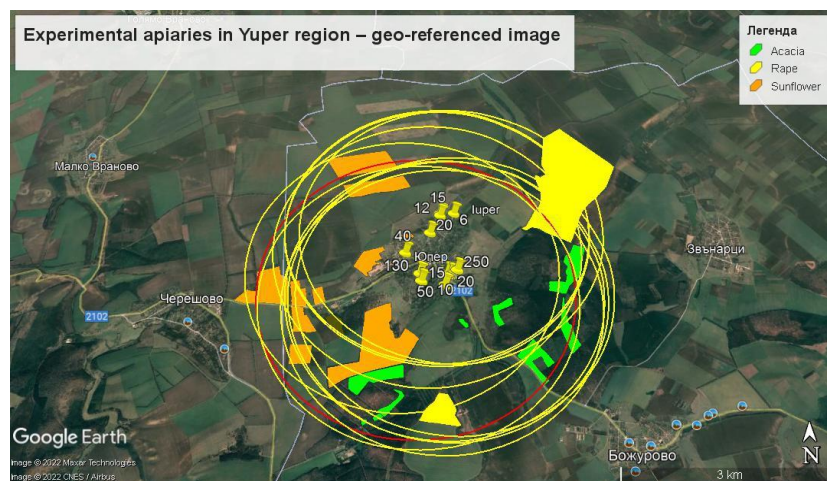

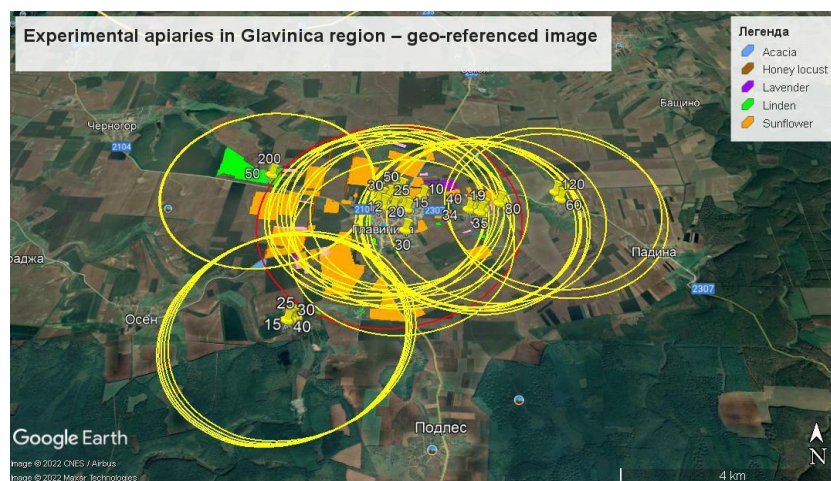Fig. 2. **Experimental apiaries in Yuper region – geo-referenced image**

Fig. 3. **Experimental apiaries in Glavinica region – geo-referenced image**

The evaluation coefficient ρ_i, preferably in a given place, the sum of the "values" of all feeding places in relation to the given apiary place is taken:

$$\rho_i = \sum_{j=1}^{n} b_j c_{ij} \frac{c_{ij}}{\sum_{k=1}^{m} c_{ij}} = \sum_{j=1}^{n} \frac{b_j c_{ij}^2}{\sum_{k=1}^{m} c_{ij}}. \tag{2}$$

Using formula (1) and (2), the 'objective' w_i^2 priority weights are also calculated for Brestovica, Yuper and Glavinica regions. The total priority weight w_i is designed as a convex linear combination of the 'subjective' w_i^1 and 'objective' w_i^2 priority weights.

$$w_i = \mu w_i^1 + (1-\mu) w_i^2. \tag{3}$$

In the present study we propose a practical technological solution for evaluation the apiaries location and three different regions spaced at a certain distance from each other with different numbers of bee colonies by application of the previous mathematical model created by us.

In the particular case, specific numbers of hives were placed on the places and the solution of the problem redistributed to the same places.

Results and discussion

Different numerical experiments were performed (at different values of $\lambda \wedge \mu$). Different solutions have been obtained. The beekeeper can choose a solution depending on the specific situation and the characteristics of the beekeeping area. The problem is solved via Matlab software operations research capabilities. Some of the results for Brestovica region are shown in Table 1 and Table 2. After the numerical experiment we calculated in Table 1 that 160 bee colonies of apiary 1; 2 bee colonies of apiary 2; 22 bee colonies of apiary 3; 18 bee colonies of apiary 4; 102 bee colonies of apiary 5; 30 bee colonies of apiary 6; 350 bee colonies of apiary 7 and 100 bee colonies of apiary 8 will be relocated in site 7 at 90% "objective" priority weights and 75% "subjective" priority weights for choosing a location. In this case the decision is multi-criterial. Interesting distribution of the relocated site at 100% priority of feeding the bees is shown in Table 2. The results in the table show that 40 and 30 bee colonies of apiaries 1 and 6 will be relocated in site 1; 2 and 102 bee colonies of apiaries 2 and 5 will be relocated in site 2; 350 bee colonies of apiary 7 will be relocated in site 2; 120 and 18 bee colonies of apiaries 1 and 4 will be relocated in site 6. The remaining 22 bee colonies of apiary 3 have to go to the site 8. The decision is one-criterial.

Table 1

Number of hives for Brestovica from the k-th apiary, that can be relocated at site i at $\lambda = 0.9$, $\mu = 0.75$ (d_k – number of hives from the k-th apiary, m – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
d_1	0	0	0	0	0	0	160	0
d_2	0	0	0	0	0	0	2	0
d_3	0	0	0	0	0	0	22	0
d_4	0	0	0	0	0	0	18	0
d_5	0	0	0	0	0	0	102	0
d_6	0	0	0	0	0	0	30	0
d_7	0	0	0	0	0	0	350	0
d_8	0	0	0	0	0	0	100	0

After calculation (X_i number of colonies to be relocated at site i, $i = 1, \grave{} m$; $X_i \in Z^+$ the distribution of the number of bee colonies and the amount of overpopulation (E_i) in Brestovica, at 100% priority of feeding the bees show that 70 bee colonies will be fully nourished at site 1; 104 bee colonies will be fed at site 2; 350 bee colonies will be fed at site 4; 138 bee colonies will be fed at site 6; 100 bee colonies will be fed at site 7; 22 bee colonies will be fed at site 8; the sites 3 and 5 will not feed any bee colonies. Their sum is 784, which is theoretically they will be fed. In this case we have not indicators for overpopulation.

Table 2

Number of hives for Brestovica from the k-th apiary, that can be relocated at site i at $\lambda = 0.1$, $\mu = 0$ (d_k – number of hives from the k-th apiary, m – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
d_1	40	0	0	0	0	120	0	0
d_2	0	2	0	0	0	0	0	0
d_3	0	0	0	0	0	0	0	22
d_4	0	0	0	0	0	18	0	0
d_5	0	102	0	0	0	0	0	0
d_6	30	0	0	0	0	0	0	0
d_7	0	0	0	350	0	0	0	0
d_8	0	0	0	0	0	0	100	0

Table 3

Number of hives for Yuper region from the k-th apiary, that can be relocated at site i at $\lambda = 0.9$, $\mu = 0.75$ (d_k – number of hives from the k-th apiary, m – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_8	m_9	m_{10}	m_{11}
d_1	0	0	0	0	0	0	1	0	249	0
d_2	0	0	0	20	0	0	0	0	0	0
d_3	0	0	0	0	0	0	0	0	10	0
d_4	0	0	0	50	0	0	0	0	0	0
d_5	0	0	0	15	0	0	0	0	0	0
d_6	0	0	0	0	0	0	0	0	130	0
d_8	0	0	0	40	0	0	0	0	0	0
d_9	0	0	0	20	0	0	0	0	0	0
d_{10}	6	0	0	0	0	0	0	0	0	0
d_{11}	0	0	0	0	0	0	0	0	12	0

From Table 3 it is clear that 20, 50 15, 40, 20 bee colonies of apiaries 2, 4, 5, 8, 9 will be relocated in site 4. The remaining 249; 130; 12 bee colonies of apiaries 1; 6; 11 have to go to the site 10 at 90% priority of feeding the bees 'objective' priority weights and 75% 'subjective' priority weights for choosing a location.

The relocated site at 100% priority of feeding the bees shows a single-criteria solution of the problem in Table 4. The results in the table show that 20 bee colonies of apiary 9 and 12 bee colonies of apiary 11 will be relocated in site 1; 50 bee colonies of apiary 4 will be relocated in site 3; 15 bee colonies of apiary 5 and 41 bee colonies of apiary 6 will be relocated in site 4. The remaining 250; 3; 89; 10 bee colonies of apiaries 1; 2; 6 and 3 have to go to the sites 6; 8; 9;11.

Table 4

Number of hives from the k-th apiary, that can be relocated at site i at $\lambda = 0.1$, $\mu = 0$ (d_k – number of hives from t the k-th apiary, m – number of relocation sites)

z_{ki}	m_1	m_2	m_3	m_4	m_5	m_6	m_8	m_9	m_{10}	m_{11}
d_1	0	0	0	0	0	250	0	0	0	0
d_2	0	0	0	0	0	0	3	0	0	0
d_3	0	0	0	0	0	0	0	0	0	10
d_4	0	0	50	0	0	0	0	0	0	0
d_5	0	0	0	15	0	0	0	0	0	0
d_6	0	0	0	41	0	0	0	89	0	0
d_8	0	0	0	0	0	0	0	0	0	0
d_9	20	0	0	0	0	0	0	0	0	0
d_{10}	0	0	0	0	6	0	0	0	0	0
d_{11}	12	0	0	0	0	0	0	0	0	0

Distribution of the number of bee colonies and the amount of overpopulation in Yuper, at 100% priority of feeding the bees show that 32 bee colonies will be fully nourished at site 1; 7 bee colonies will be fed at site 2; 50 bee colonies will be fed at site 3; 56 bee colonies will be fed at site 4; 14 bee colonies will be fed at site 5; 250 bee colonies will be fed at site 6; 3 bee colonies will be fed at site 7; 69 bee colonies will be fed at site 8; 10 bee colonies will be fed at site 10 and 57 bee colonies will be fed at site 11. The site 9 will not feed any bee colony. Their sum is 548, which is theoretically they will be fed. It was found that in this case we have overpopulation. The amount of overpopulation in site i is 20. Similar results are obtained for Glavinica region. Their sum of bee colonies is 486, which is theoretically they will be fed. It was found that in this case we have overpopulation. The amount of overpopulation in site i is 739.

The obtained results show that we will not have overpopulation only for the Brestovitca region and distribution of colonies that can be fed is optimal according to the relevant criteria with selected parameters $\lambda \wedge \mu$. Regardless of which solution the beekeeper chooses, each of them is applicable and bee colonies will be fed. For the other two regions, Yuper and Glavinitca there will not be complete nutrition for bee colonies regardless of the redistribution of bee colonies in individual potential places, the problem of overpopulation cannot be solved.

Conclusions

The mathematical model used to evaluate apiaries' location is suitable for flat-hilly regions with and without overpopulation. The model allows optimal distribution of bee colonies in the designed places depending on the food resources in the studied regions. For the three studied areas it was found that with the greatest potential for feeding the bee colonies is Brestovitca, where we have no overpopulation. With small overpopulation of 20 bee colonies is the region of Yuper. The highest overpopulation of 739 bee colonies is observed in the area of Glavinica.

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Author contributions

Conceptualization, A.A.; methodology, A.A. and I.G.; software, I.G.; validation, A.A.; formal analysis, A.A and I.G..; investigation, A.A., I.H., P.H.; data curation, A.A., I.G.; writing – original draft preparation, A.A.; writing – review and editing, A.A., I.H. and P.H.; visualization, A.A.; project administration, A.A.; funding acquisition, A.A. All authors have read and agreed to the published version of the manuscript.

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